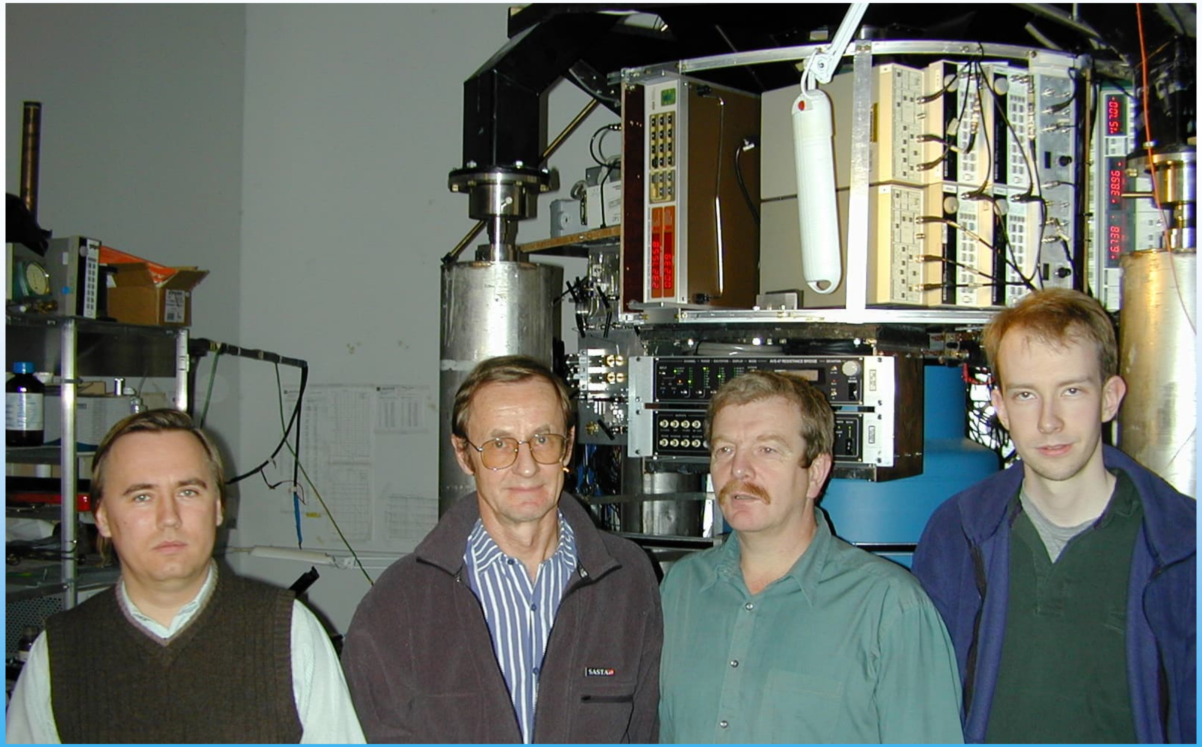
FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

On Transition to Quantum Turbulence in Helium Superfluids

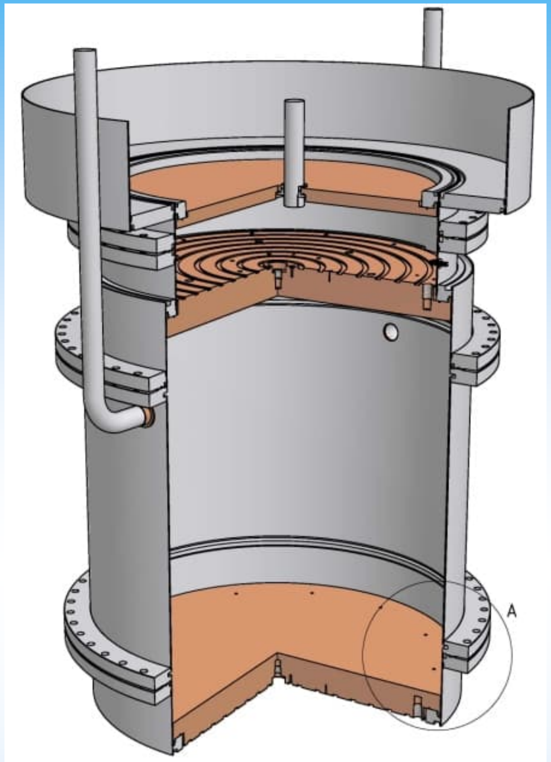
L. Skrbek

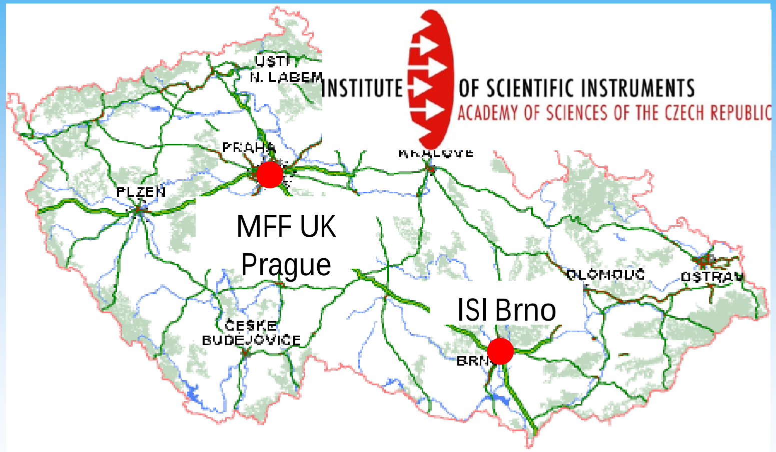
Faculty of Mathematics and Physics, Charles University,
Ke Karlovu 3, 121 16 Prague, Czech Republic



<Matti Krusius 80>
Otaniemi 2022

Experimental RBC apparatus – ISI Brno





Copper plates (OFHC) heat conductivity (at 5 K)

2 kW/m/K (RRR~300)

Thermally fast RBC cell

PHYSICAL REVIEW LETTERS **128,** 134502 (2022)

Editors' Suggestion | Featured in Physics

Thermal Waves and Heat Transfer Efficiency Enhancement in Harmonically Modulated Turbulent Thermal Convection

P. Urban,[1] P. Hanzelka,[1] T. Králík,[1] V. Musilová,[1] and L. Skrbek[2]

Propagation and Interference of Thermal Waves in Turbulent Thermal Convection

P. Urban,[1] T. Králík,[1] V. Musilová,[1] D. Schmoranzer,[2] and L. Skrbek[2]

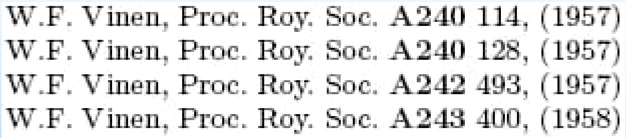
Under review PRL

Quantum turbulence occurs in quantum fluids

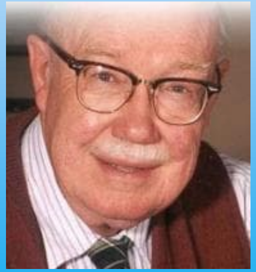
- Quantum fluids are so called because their physical properties cannot be explained by classical physics, they depend on quantum physics
- Quantum fluids (such as two stable isotopes of liquid helium at very low temperature) display superfluidity and two-fluid behaviour
- Quantum turbulence can be defined loosely as the most general way of motion of a quantum fluid displaying superfluidity

Historically, QT in He II was mentioned as a theoretical possibility by R. P. Feynman, who recognized that QT ought to take the form of a random tangle of quantized vortices.
`` Application of quantum mechanics to liquid helium",
Prog. in Low Temp. Phys., vol. 1, (1955)

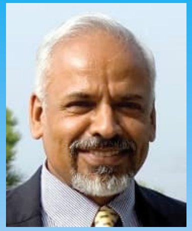
Experiment - thermal counterflow in He II- a form of motion peculiar to two-fluid superfluid hydrodynamics –was first investigated by W. F. Vinen
(passed away on June 8, 2022)



W.F. Vinen, Proc. Roy. Soc. A240 114, (1957)
W.F. Vinen, Proc. Roy. Soc. A240 128, (1957)
W.F. Vinen, Proc. Roy. Soc. A242 493, (1957)
W.F. Vinen, Proc. Roy. Soc. A243 400, (1958)



The term Quantum Turbulence was introduced into the literature by C. F. Barenghi in his PhD. Thesis (1982) and later by R. J. Donnelly in a symposium dedicated to G. I. Taylor
R. J. Donnelly , C. E. Swanson, ``*Quantum turbulence,"* J. Fluid Mech. 173, 387 (1986)



Various forms and regimes of QT exist in helium superfluids

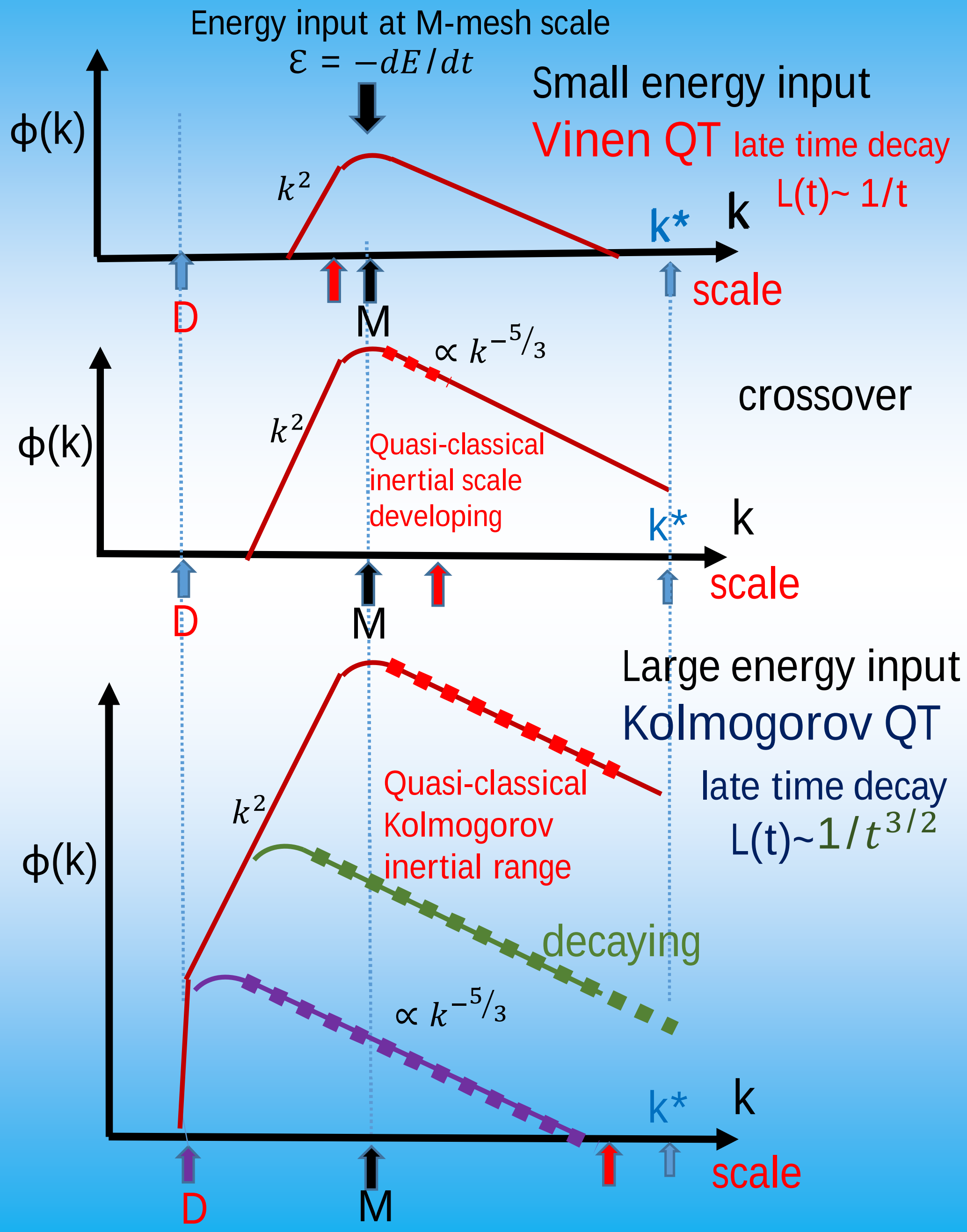
(C.F. Barenghi, LS, Katepalli R. Sreenivasan,
Quantum turbulence,
Cambridge University Press, in print)

- A key point: for classical turbulence in the unbounded case, there are only two length scales to consider: M and

$$\eta \approx (\varepsilon/\nu^3)^{-1/4}$$

- In superfluid turbulence, there is an additional important scale, quantum length scale $\ell_Q \approx (\varepsilon/\kappa^3)^{-1/4}$
- Even pure superfluid turbulence in the zero T limit, therefore cannot be generally considered, in contrast to statements in the literature, a simple “prototype” of turbulence.
- Despite the similarities, QT is different and more complex than the classical case.

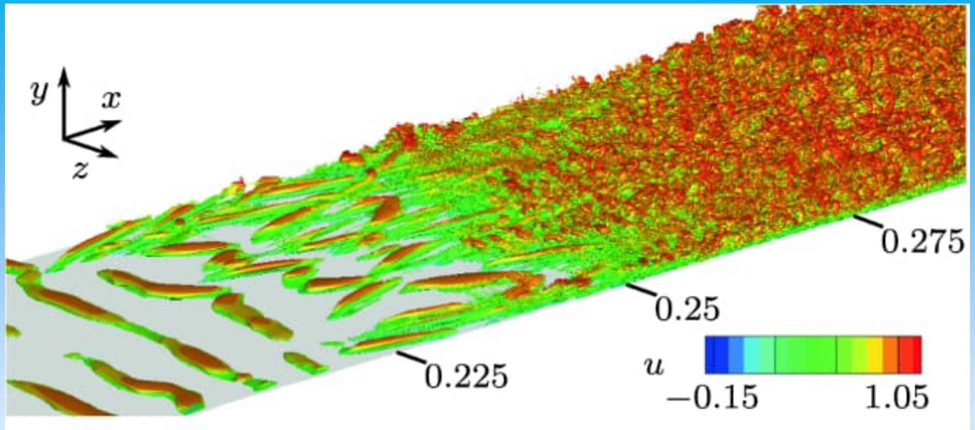
Superfluid energy spectrum in zero T limit



Transition to turbulence in classical fluids,

generally occurs upon exceeding some critical parameter, such as Re

- growth of perturbations of an underlying basic laminar flow
- a complicated process



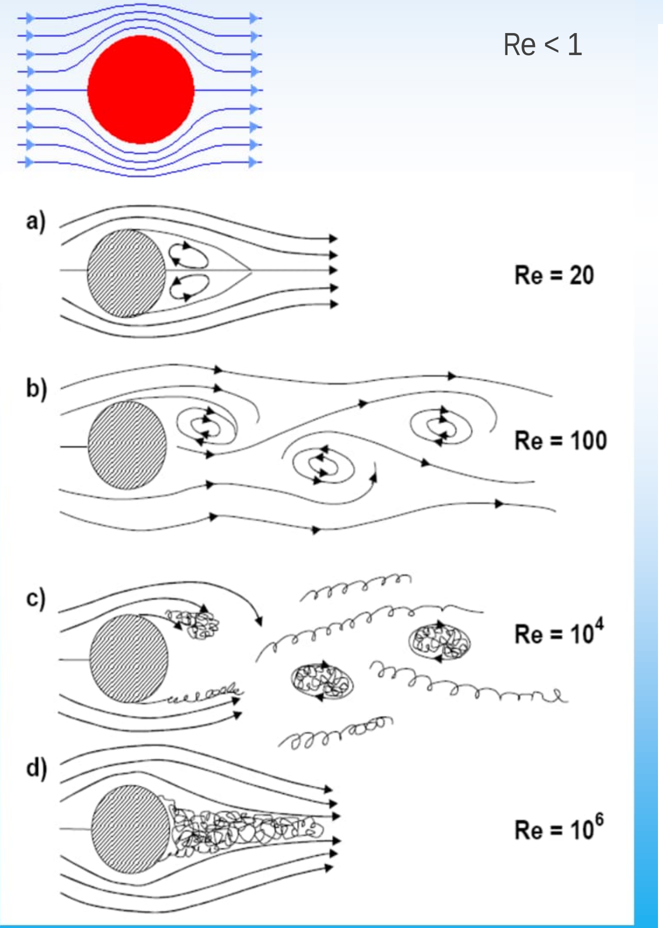
Laminar flow

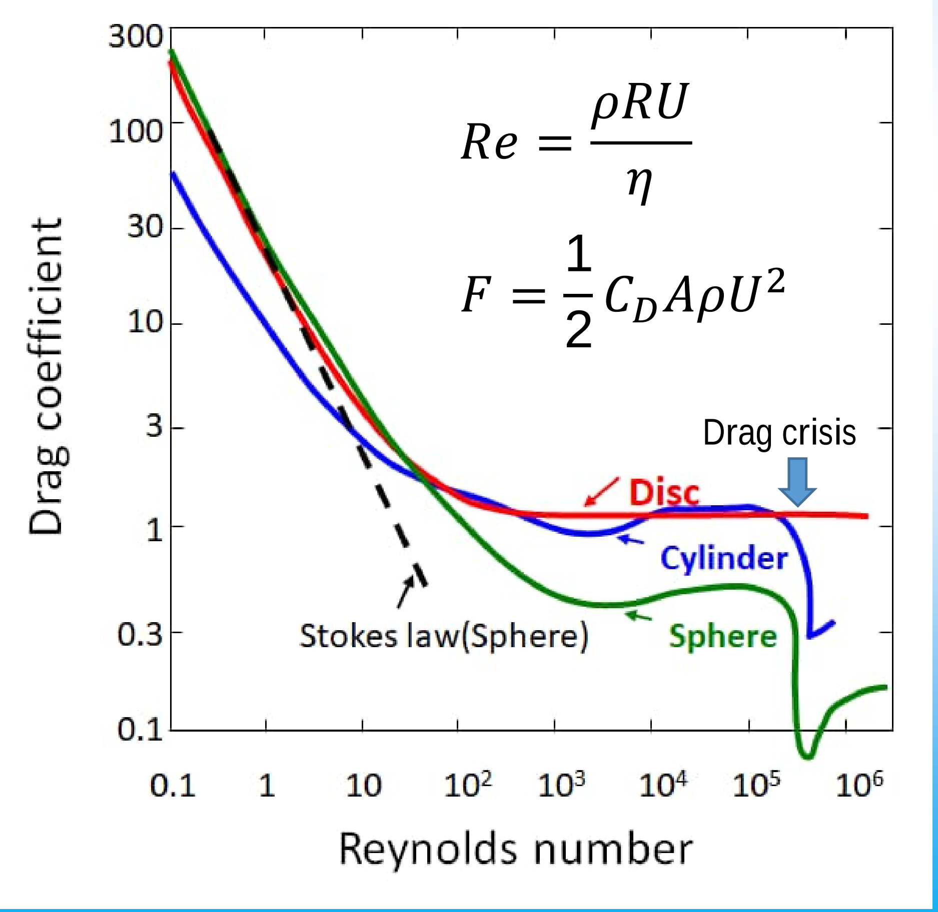
Boundary layer separation, laminar wake

Kármán vortex street

Turbulence in the wake

Turbulent boundary layer, drag crisis





$$Re = \frac{\rho R U}{\eta}$$

$$F = \frac{1}{2} C_D A \rho U^2$$

F - drag force
C_D - drag coefficient
U - velocity
A - projected area ⊥ flow
ρ - density
R - characteristic dimension
η - dynamic viscosity

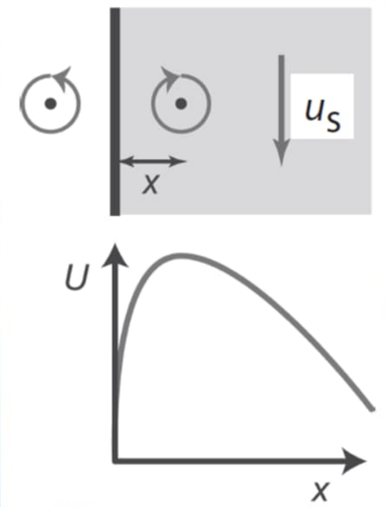
Transition to quantum turbulence, especially in the two-fluid nature at finite T is even more complex

Superfluid turbulence = turbulence in the superfluid component,
Involves quantized vortices

- Complicated process of vortex nucleation
- Quantized vortices appear spontaneously while cooling helium through Tc (Kibble-Zurek mechanism)
- They decay and disappear at lower temperature.
- Rough walls of the vessel - pinning centres to anchor remnant vortices.

D.D. Awschalom, K.W. Schwarz, *PRL.* 52, 49 (1984).

Image vortex



potential barrier opposing vortex nucleation

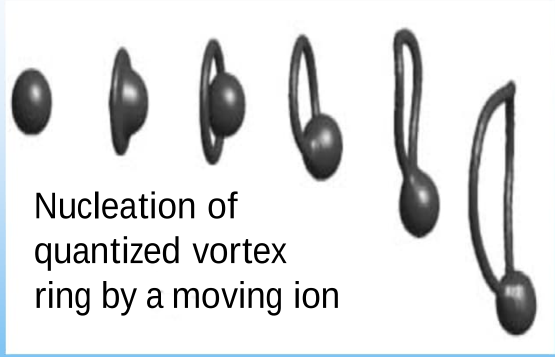
Vortex nucleation

Intrinsic

He II crit. velocity≈ 10 m/s, large enough to make it unlikely

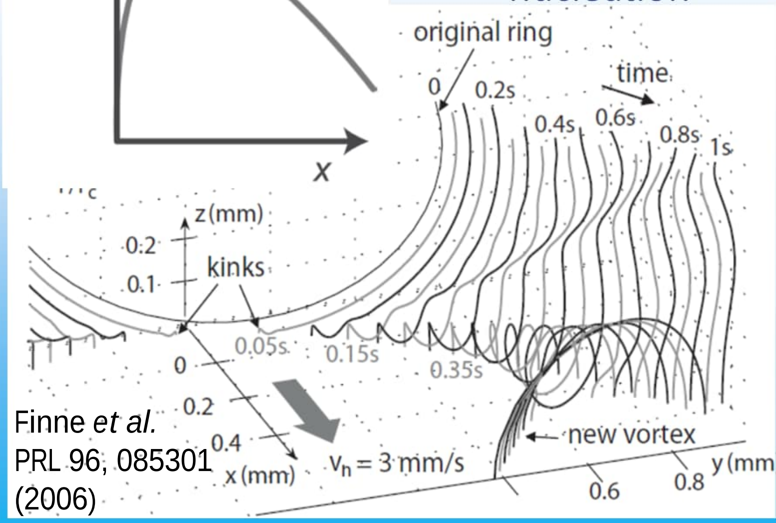
Extrinsic

He II crit. velocity≈ cm/s



Nucleation of quantized vortex ring by a moving ion

3He-B both intrinsic and extrinsic nucleation is possible (smooth walls in comparison with the vortex core size)



Finne *et al.*
PRL 96, 085301 (2006)

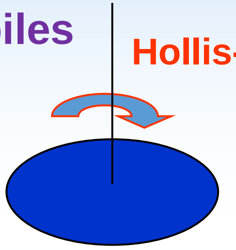
Transitions leading to steady-state quantum turbulence

- conceptually simplest case T ⟶ 0 : transition to quantum turbulence from steady potential superflow
- experimentally the most difficult: realization of steady potential flow is nearly impossible to achieve
- in experiments therefore: mostly flows due to oscillating objects
- finite T: complicated case due to two-fluid behavior, transition can occur in either component

Examples of oscillating objects used in experiments in He II (and in ^{3}He)

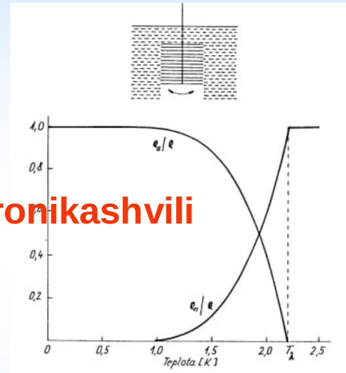
Discs and piles of discs

Torsional oscillators

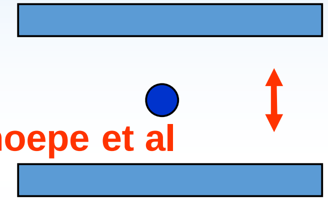


Hollis-Hallett

Andronikashvili

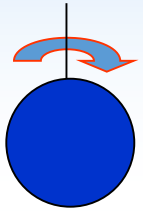


Spheres



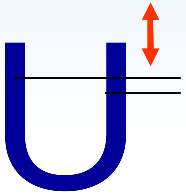
Schoepe et al

Luzuriaga Hollis-Hallett

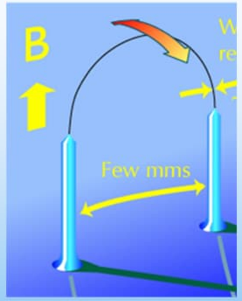


U-tubes

Donnelly Penrose



Wires, tuning forks

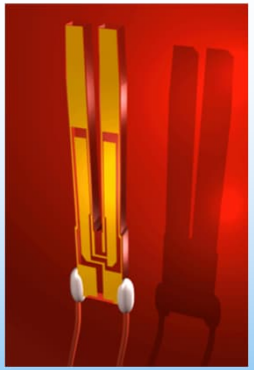


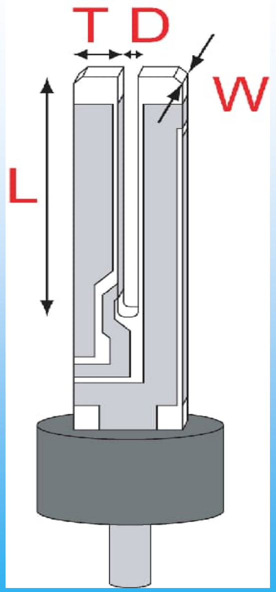
Many authors – vibrating wire viscometers

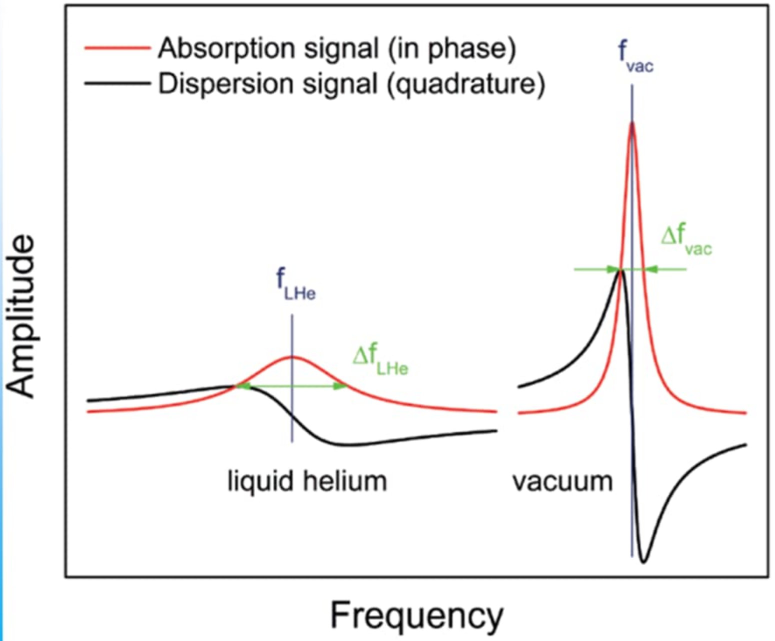
Vinen

Morishita, Kuroda, Sawada, Satoh, JLTP 76, 387 (1989)

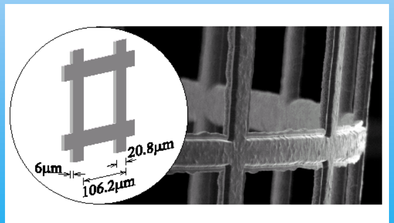
Lancaster, Osaka, Kosice, Moscow, Helsinki, Grenoble, Prague………

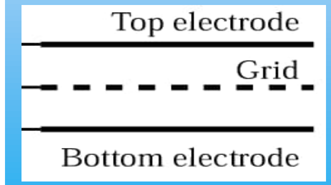


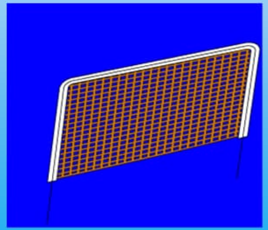




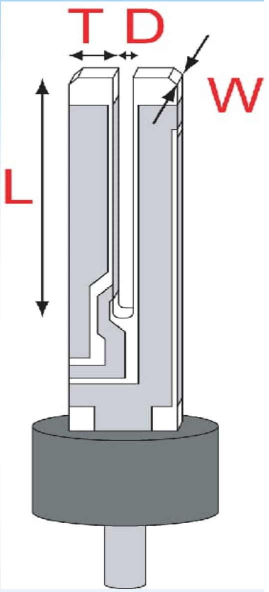
Grids

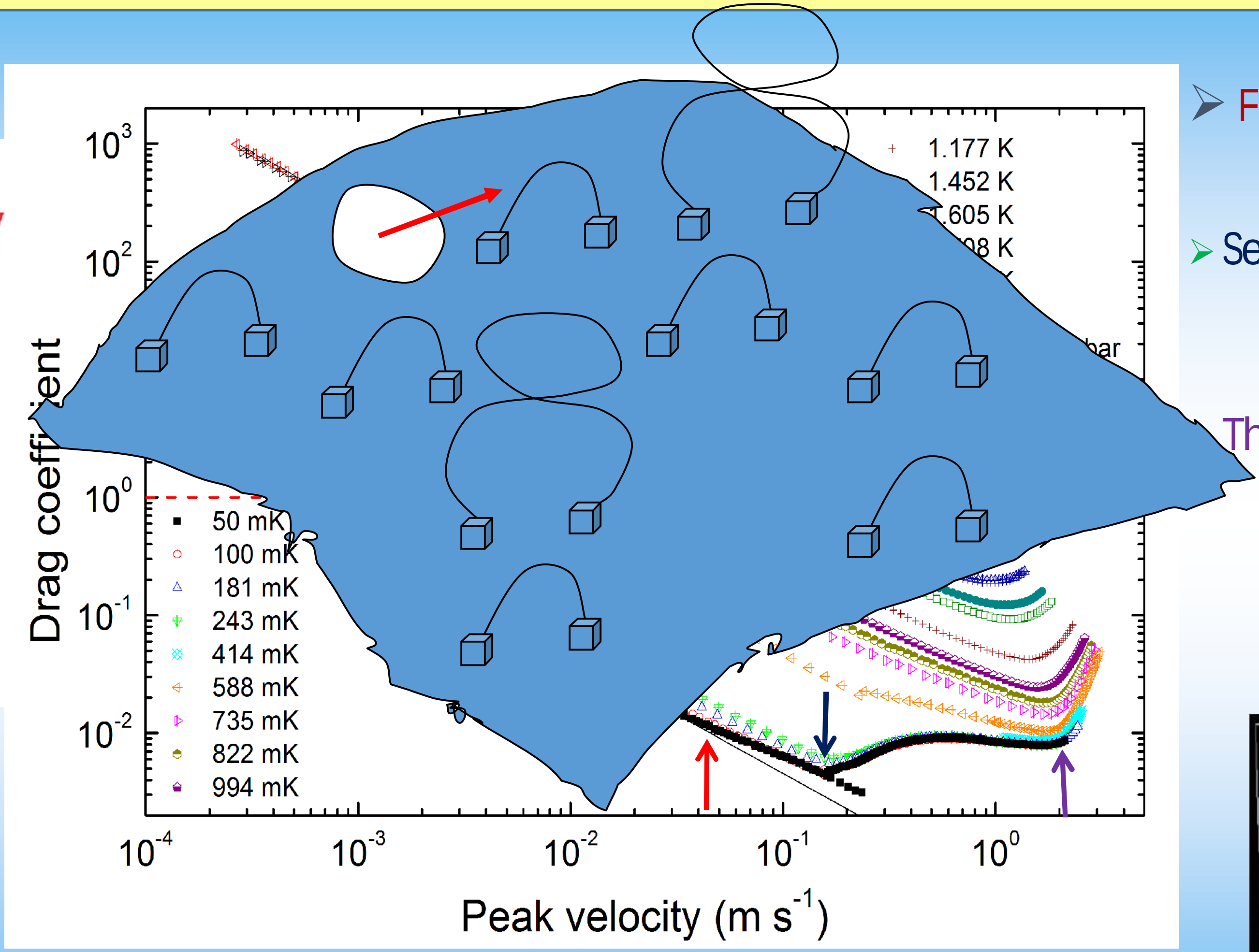






In He II (at low enough T) drag coefficient displays three critical velocities

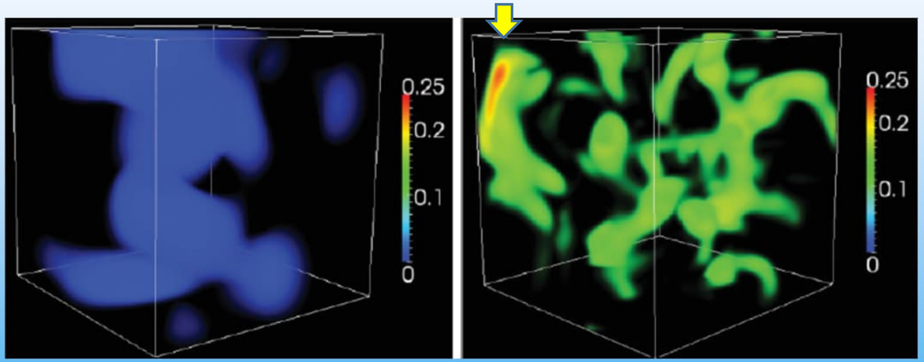




- **First critical velocity** – changes in frequency, little effect on drag force
  - effective mass rises due to vortices pinned on the oscillator surface
  - need not lead to increased drag (mostly potential flow)
- **Second critical velocity** – non-linear dissipation sets in
  - vortices spread into the bulk, carrying energy & momentum away
  - if $C_D \ll 1$ – the "wake" past the oscillator is not classical-like, no large structures in the flow, building up the vortex tangle
- **Third critical velocity** – large structures start to develop in the tangle
  - drag rises towards classical value (full pressure drag, developed wake)

Vinen turbulence
featureless

Kolmogorov turbulence
intense vortical region



Coarse-grained superfluid vorticity

L.K. Sherwin-Robson, C.F. Barenghi, A.W. Baggaley, *PRB* 86, 104501 (2012)

For details, see: D. Schmoranzer, M.J. Jackson, V. Tsepelin, M. Poole, A.J. Woods, M. Clovecko, LS: Multiple Critical Velocities in Oscillatory Flow of Superfluid 4He due to Quartz Tuning Forks PRB 94, 214503 (2016)

Oscillatory flows of viscous fluids

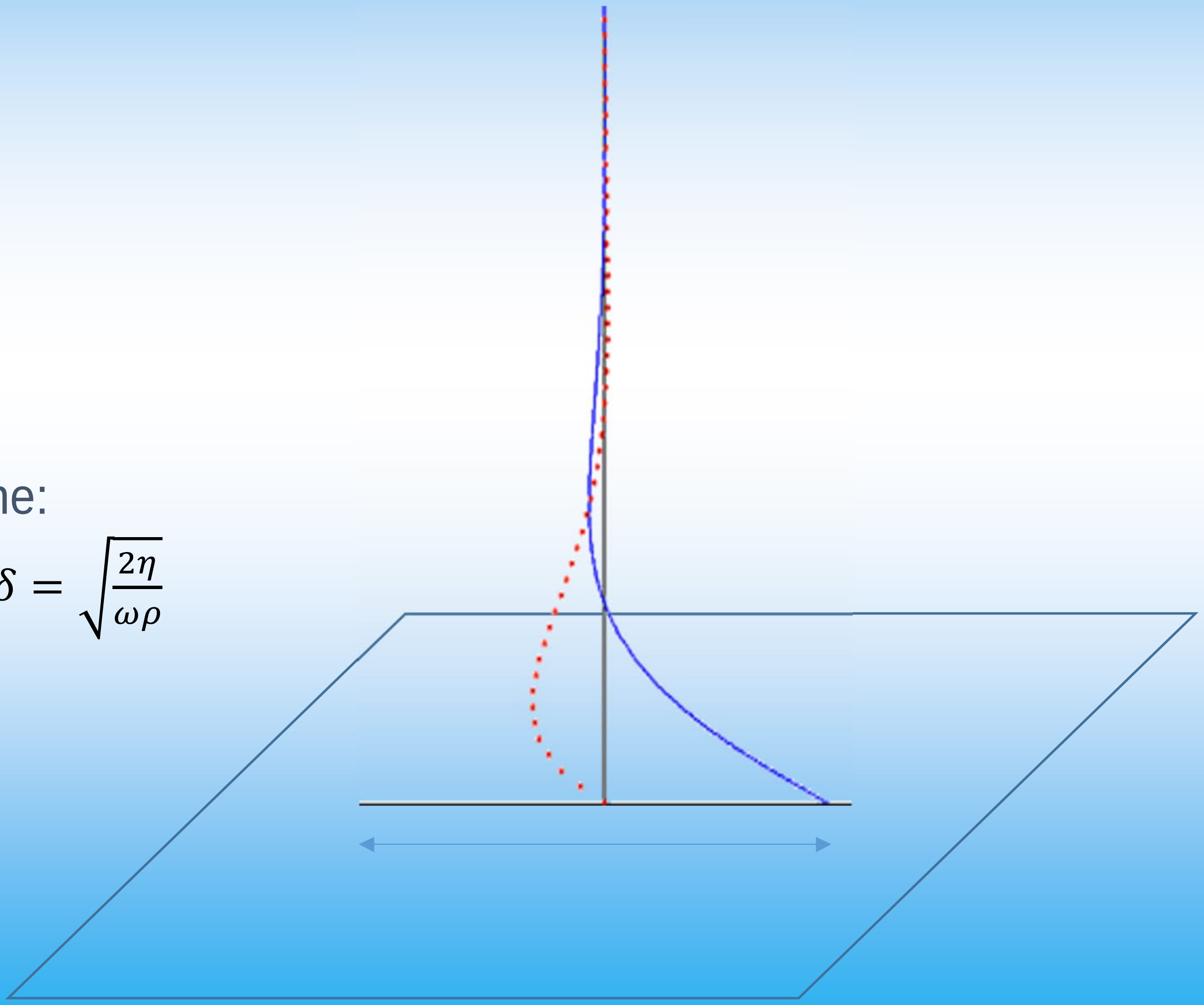
oscillatory flow of a bluff body of size *D* is fully described by two dimensionless parameters, such as, e.g.,

Stokes number $$St = \frac{D^2}{\pi\delta^2}$$

Reynolds number $$\mathrm{Re} = \frac{DU}{\nu}$$

Model system - oscillating plane:

Stokes boundary layer thickness: $\delta = \sqrt{\frac{2\eta}{\omega\rho}}$



Dimensionless governing equation

Introducing dimensionless quantities (velocity amplitude, characteristic length scales and *independent timescale* – angular frequency of oscillations):

$\mathbf{u} = U\mathbf{u}'; \ L_i\nabla = \nabla'; \ \omega t = t'; \ p = p'\rho U^2$ leading to the Navier-Stokes Equation of the form

$$\omega U \frac{\partial \mathbf{u}'}{\partial t'} + \frac{U^2}{L_1}[(\mathbf{u}' \cdot \nabla')\mathbf{u}' + \nabla' p'] = \frac{\eta U}{\rho {L_2}^2}\Delta'\mathbf{u}'$$

Geometry and surface roughness matter:

Classical high Stokes number oscillatory flows

$$St = \frac{D^2}{\pi\delta^2} >> 1$$

Body size D, dynamic viscosity η

Stokes boundary layer thickness (viscous penetration depth)

$$L_1 = L_2 = \delta = \sqrt{2\eta/(\rho\omega)} << D$$

In this case, Navier-Stokes equation may be expressed using only one dimensionless parameter

Boundary-layer-based Reynolds number

$$Dn = \mathrm{Re}_\delta = \delta\rho U/\eta$$

Dimensionless form of NSE: $2\frac{\partial \mathbf{u}'}{\partial t} + Dn[(\mathbf{u}' \cdot \nabla)\mathbf{u}' + \nabla' p'] = \Delta'\mathbf{u}'$

Instabilities and transition to QT in high-Stokes-number oscillatory flows of He II

- Above about 1K, He II displays the two-fluid behavior
- Isothermal flow at low velocities (with no quantized vortices present) - two independent velocity fields
- Instability can occur either in normal fluid or in superfluid

Normal fluid

In analogy with classical high Stokes number oscillatory flows

- viscous penetration depth of the normal fluid

$$\delta_n = \sqrt{2\eta/(\rho_n\omega)} << D$$

- Define Donnelly number $Dn = \delta_n \rho_n U / \eta$
- R. J. Donnelly and A. C. Hollis-Hallett, Ann. Phys. 3, 320 (1958)
- drag coefficient related to laminar flow

$$C_D^n = \frac{\Phi}{Dn}$$

- Instability occurs upon exceeding a critical value of Dn_{cr}

Superfluid

Instability occurs upon exceeding a critical velocity U_{cr} due to Donnelly-Glaberson instability leading to the production of quantized vorticity

D.K. Cheng, M.W. Cromar, R.J. Donnelly, PRL 31, 433 (1973)

W. I. Glaberson et al., PRL 33, 1197 (1974), R.M. Ostermeier, and W.I. Glaberson, 21, 191 (1975)

Which instability occurs first?

Is a (T-dependent) cross-over possible?

The Donnelly number

$$Dn \equiv \frac{\delta_n \rho_n U}{\eta}$$

ANNALS OF PHYSICS: 3, 320-345 (1958)

Periodic Boundary Layer Experiments in Liquid Helium*

R. J. DONNELLY

Institute for the Study of Metals and Department of Physics, University of Chicago, Chicago, Illinois

AND

A. C. HOLLIS HALLETT

Department of Physics, University of Toronto, Toronto, Ontario

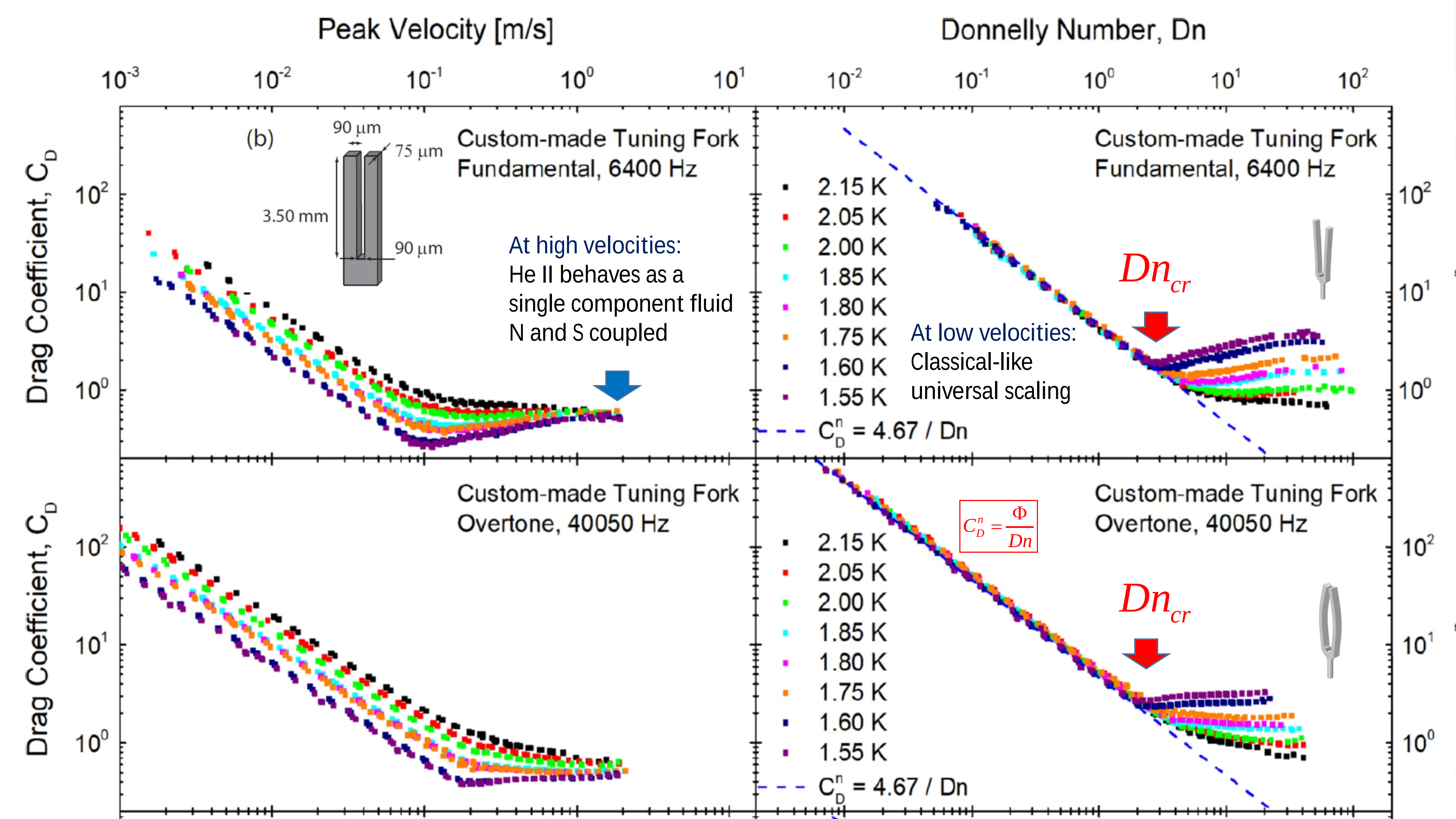
Defined originally for a smooth torsionally oscillating sphere with periods of 6.5 s < T < 25.0 s, but failed to describe the onset of nonlinear dissipation and hence was abandoned.

It is, however, valid for *laminar drag force scaling* and even the *instabilities* in the case of hydrodynamically rough objects.

which it has in the whole region A. Since there is no indication of any discontinuous change in either decrement or period of oscillation at ϕ_c, then, at ϕ_c, the penetration depth must be equal to λ_n. Similarly the kinematic viscosity should be $\nu_n (\equiv \eta_n/\rho_n)$. Thus we should form a Reynolds number defined as

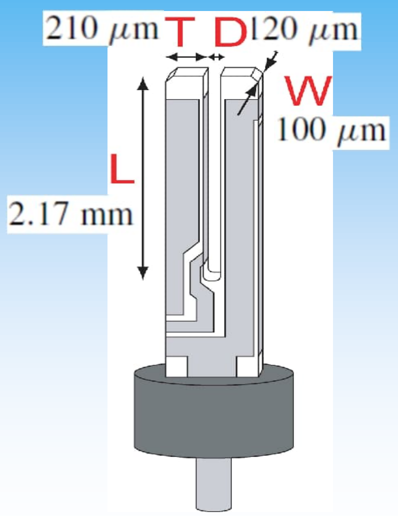
$$\mathcal{R}_n = (\omega R \phi) \cdot \lambda_n / \nu_n . \qquad (24)$$

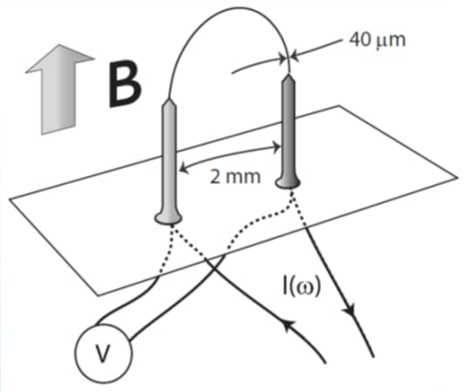
$\lambda_n (= \sqrt{2\eta_n/\rho_n \omega})$

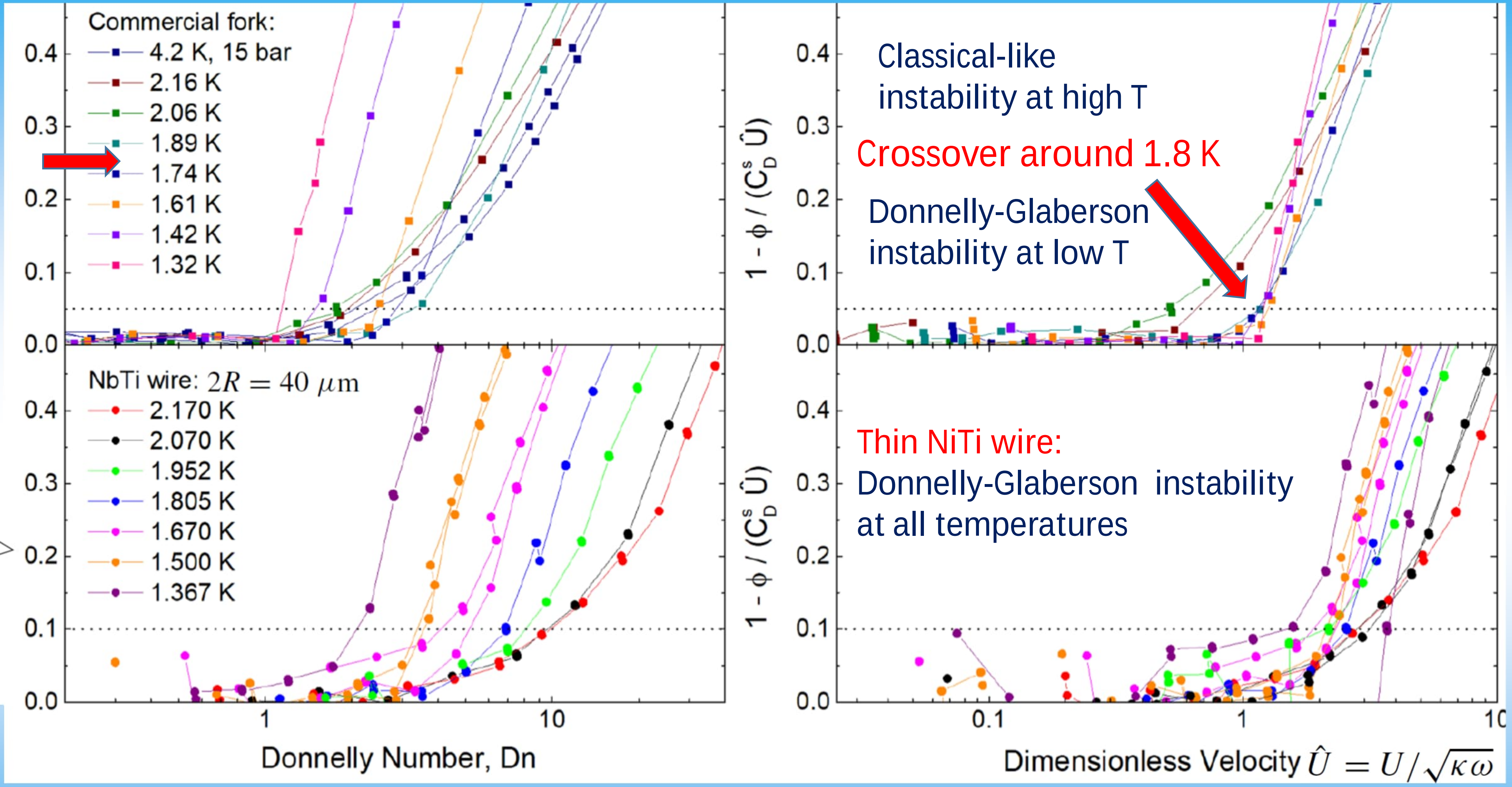


Peak Velocity [m/s]
Donnelly Number, Dn
Drag Coefficient, C_D
(b)
90 μm
75 μm
3.50 mm
90 μm
Custom-made Tuning Fork
Fundamental, 6400 Hz
At high velocities:
He II behaves as a single component fluid
N and S coupled
2.15 K
2.05 K
2.00 K
1.85 K
1.80 K
1.75 K
1.60 K
1.55 K
C_D^n = 4.67 / Dn
Dn_{cr}
At low velocities:
Classical-like universal scaling
Custom-made Tuning Fork
Overtone, 40050 Hz
$C_D^n = \frac{\Phi}{Dn}$

Further analysis, calculate the dimensionless nonlinear force







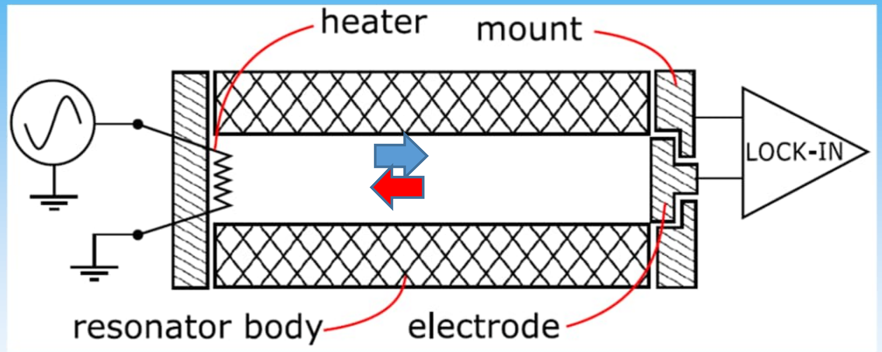
crossover between classical-like and Donnelly-Glaberson instabilities observed, thanks to the steep temperature dependence of the kinematic viscosity of the normal fluid

D. Schmoranzer, M. J. Jackson, Š. Midlik, M. Skyba, J. Bahyl,T. Skokankova, V. Tsepelin, LS: PRB 99, 054511 (2019)

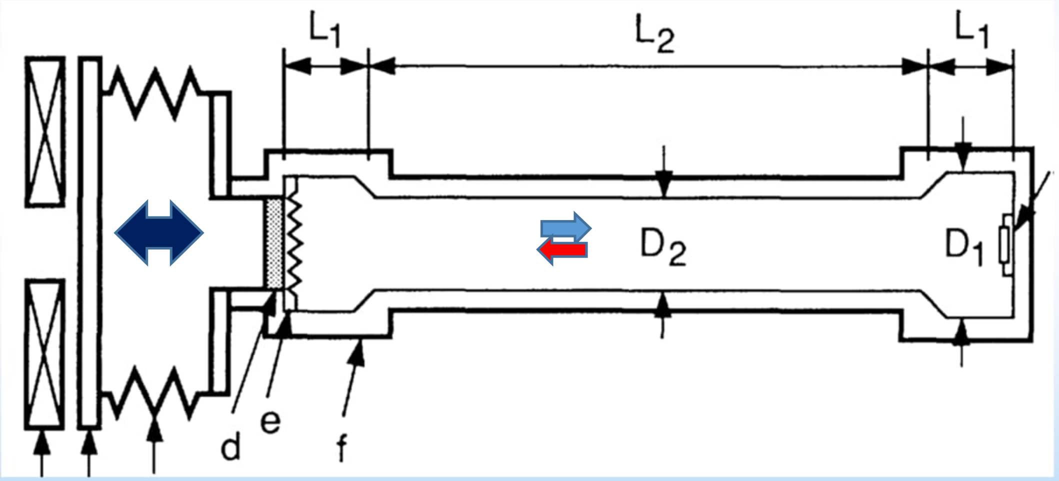
Transition to quantum turbulence in oscillatory (ac) thermal counterflow of He II

QT is generated by second sound

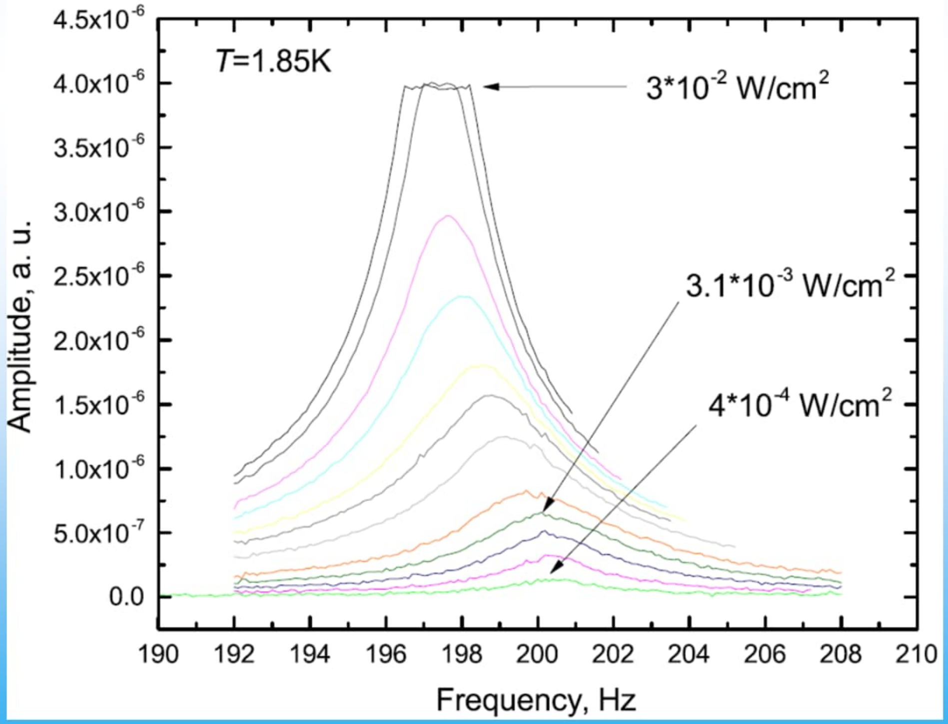
mechanically thermally



T. V. Chagovets, Physica B 488, 62 (2016)



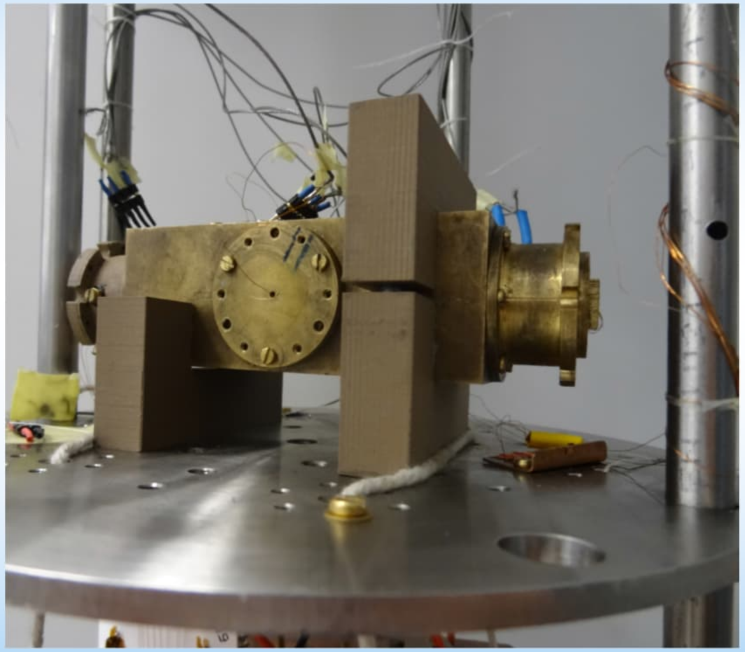
V. Kotsubo, G.W. Swift, PRL 62, 2604 (1989);
JLTP 78, 351 (1990)

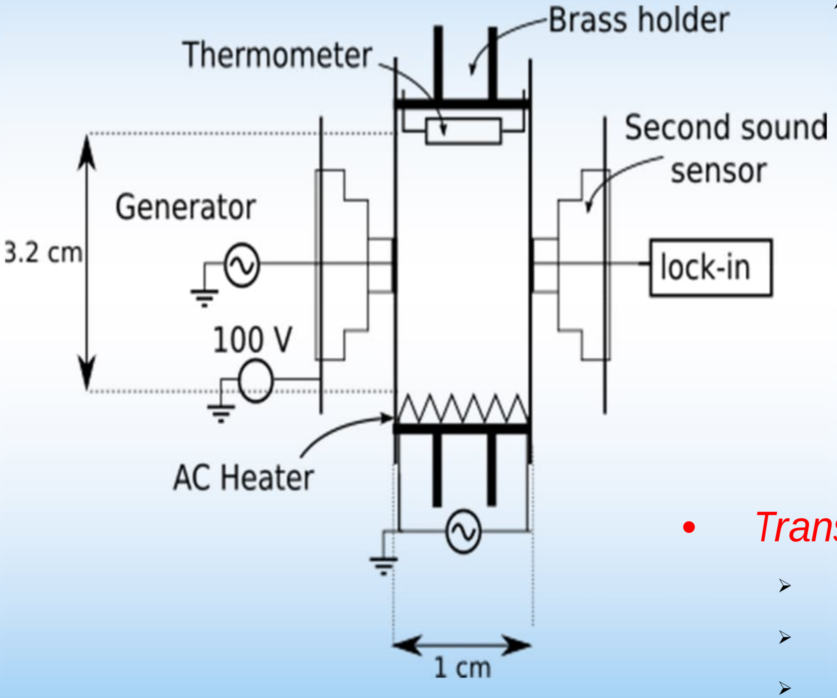


Flat top Lorentzian response

Experimental setup

- Brass channel blocked at both ends (L = 32 mm, D = 10 mm)
  - Resistive heater and thermometer at ends
  - Nuclepore membrane second sound sensors in the middle



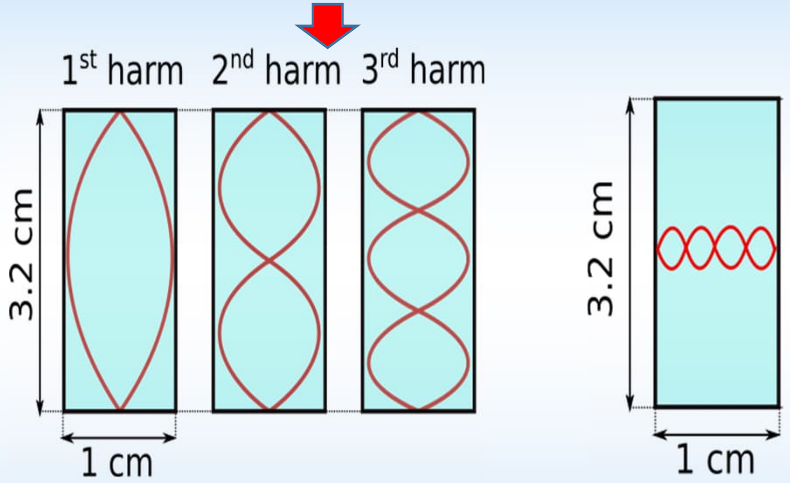


S. Midlik, D. Schmoranzer, and LS: PRB 103, 134516 (2021).

- *Longitudinal second sound resonances*

- *Low drives:* Lorentzian resonances
- *High drives:* flat-top peaks
  - Energy spent to produce turbulence

- *Longitudinal second sound modes*
  - Standing wave driven by heater (AC voltage)
  - Detection: miniature Ge/GaAs thermometer
  - High amplitude -> *turbulence generation*
  - First three harmonic modes observed



- *Transversal second sound mode*
  - Two capacitive sensors in the center of channel
  - Low amplitude used for *turbulence detection*
  - Normal component *interacts with quantized vortices*
  - Attenuation in the presence of quantum turbulence
  - Local determination of vortex line density

$$L(t) = \frac{6\pi\Delta f_0}{B\kappa}\left(\frac{A_0}{A(t)} - 1\right)$$

Assumes isotropic vortex tangle...

Hänninen and Schoepe:

JLTP 153, 189 (2008; 158, 410 (2010)

onset of quantum turbulence in oscillatory flows of superfluid helium is universal, and can be derived from a general argument based on the "superfluid Re".

$$U_{cr}^{s} \approx \sqrt{8\kappa\omega/\beta}$$

κ is the circulation quantum
β parameter of order unity

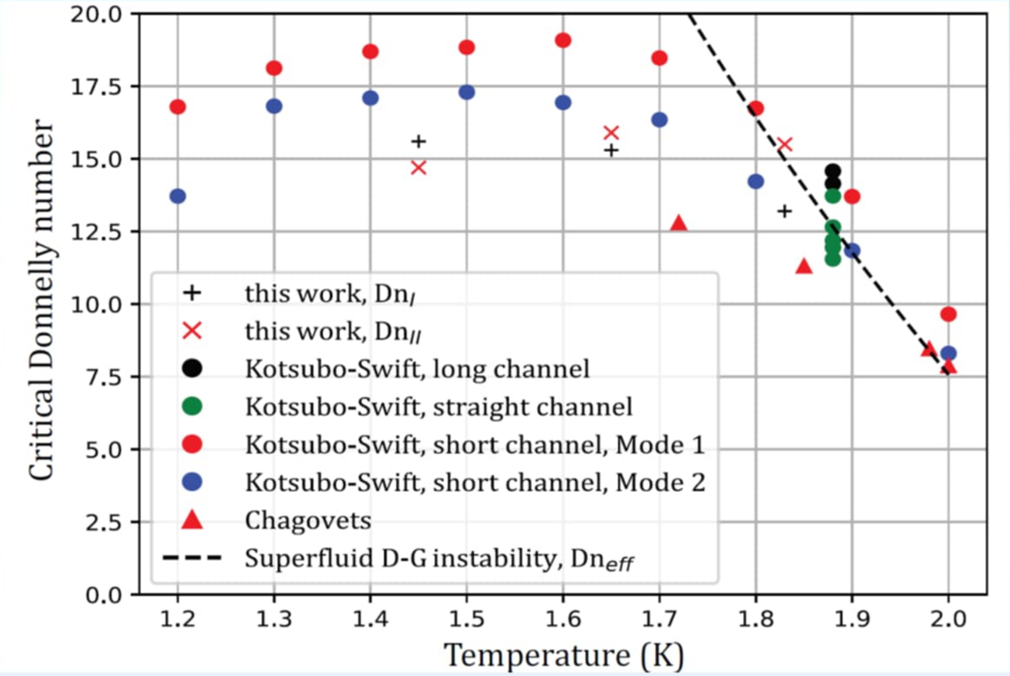
Slow increase with T, fair agreement with oscillating sphere, but disagrees with ac counterflow below about 1.8 K
V. Kotsubo, G.W. Swift, PRL 62, 2604 (1989); JLTP 78, 351 (1990)

For ac counterflow, using $U^{s}\rho_{s} = U^{n}\rho_{n}$
we calculated the critical Donnelly number

- Below 1.8 K the classical-like instability in the normal fluid occurs first
- Above 1.8 K, the Donnelly-Glaberson instability in the superfluid occurs first

classical-like instability in NF

DG inst. in SF



S. Midlik, D. Schmoranzer, and LS: PRB 103, 134516 (2021).

Note: In ac coflow $U^{s} = U^{n}$
In ac counterflow $U^{s}\rho_{s} = U^{n}\rho_{n}$

The crossovers occur in opposite sense with temperature !!!

Summary – Transition to QT in oscillatory flows of He II

At low enough *T*, three critical velocities have been observed and phenomenologically understood

At *T* >1K, in the two-fluid region, situation is more complex, as two independent velocity fields exist in isothermal flows at low velocities (with no quantized vortices)

- We have shown that for high Stokes number oscillatory flows instability can occur
- either in the normal fluid (upon reaching a critical Donnelly number)
- or in the superfluid (upon reaching critical velocity for Donnelly-Glaberson instability)

- Crossover between the two scenarios is observed
- In flow due to an oscillating objects (DG instability at low temperature)
- In second sound (DG instability at high temperature)

Transitions to QT in high Stokes flows are, on phenomenological level, understood